Statistical Analysis of Newborn Male Infants' Birth Weights

1 Introduction

A sample of newborn male infants had their birth weight measured in ounces. The data was then grouped into classes, where Z_k represents the class midpoint and N_k represents the frequency of the k-th class.

2 Number of Individuals

Let the number of classes be:

$$K = 15 \tag{1}$$

Then the total number of individuals is:

$$n = \sum_{k=1}^{K} N_k = 2 + 6 + \cdots + 1 = 9,465. \tag{2}$$

Answer

- $n = 9,465$

3 Mean and Variance

Let the individual values be denoted as:

$$x_i, \quad i \in \{1, 2, \ldots, n\}. \tag{3}$$

The relative frequency is given by:

$$p_k = \frac{N_k}{n}. \tag{4}$$

The mean is computed as:

$$\bar{x} = \sum_{k=1}^{K} p_k Z_k = \frac{1}{n} \sum_{k=1}^{K} N_k Z_k = \frac{1,035,467}{9,465} \approx 109.400. \tag{5}$$

The variance is given by:

$$\sigma^2 = \sum_{k=1}^{K} p_k (Z_k - \bar{x})^2 = \frac{1}{n} \sum_{k=1}^{K} N_k (Z_k - \bar{x})^2 = \frac{1,748,957}{9,465} \approx 184.782. \tag{6}$$

Answer

- $\bar{x} = 109.400$
- $\sigma^2 = 184.782$

4 Conversion to Grams

Each individual weight in ounces is converted to grams:

$$y_i = 28.349 \cdot x_i. \tag{7}$$

The new mean and variance become:

$$\bar{y} = 28.349 \cdot \bar{x} = \frac{29,354,453,983}{9,465,000} \approx 3,101.369, \tag{8}$$

$$\sigma_y^2 = 28.349^2 \cdot \sigma^2 = \frac{140,557,692,831}{946,500} \approx 148,502.581. \tag{9}$$

Answer

- $\bar{y} = 3,101.369$
- $\sigma_y^2 = 148,502.581$

5 Quartiles and Median

The index of the median is:

$$i_m = \frac{n+1}{2} = 4,733. \tag{10}$$

The median class satisfies:

$$\sum_{k=1}^{6} N_k = 3,049 < i_m < 5,289 = \sum_{k=1}^{7} N_k. \tag{11}$$

The median is calculated as:

$$m = Z_6 + (Z_7 - Z_6)\left(\frac{i_m - \sum_{k=1}^{6} N_k}{N_7}\right) \approx 105.014. \tag{12}$$

Similarly, the first and third quartiles are computed using:

$$i_{Q_1} = \frac{n}{4} + \frac{1}{2} = 2,366.75, \tag{13}$$

$$i_{Q_3} = \frac{3}{4}n + \frac{1}{2} = 7,099.25. \tag{14}$$

The quartile classes satisfy:

$$\sum_{k=1}^{5} N_k = 1,320 < i_{Q_1} < 3,049 = \sum_{k=1}^{6} N_k, \tag{15}$$

$$\sum_{k=1}^{7} N_k = 5,289 < i_{Q_3} < 7,296 = \sum_{k=1}^{8} N_k. \tag{16}$$

The first and third quartiles are given by:

$$Q_1 = Z_5 + (Z_6 - Z_5)\left(\frac{i_{Q_1} - \sum_{k=1}^{5} N_k}{N_6}\right) \approx 96.449, \tag{17}$$

$$Q_3 = Z_7 + (Z_8 - Z_7)\left(\frac{i_{Q_3} - \sum_{k=1}^{7} N_k}{N_8}\right) \approx 114.216. \tag{18}$$

Answer

- $m_p = 105.014$
- $Q_1 \approx 96.449$
- $Q_2 = 105.014$
- $Q_3 \approx 114.216$