Question 3: Statistical Analysis of Test Grades

Part 1: Frequency Distribution

The histogram representing the test grades distribution is shown below:

Stem	Leaf
5	2
6	2 8
7	1 5 9
8	1 6 6 6 8 9
9	2 3 3 5 6 8
10	0 0

Answer: Histogram shown above.

Part 2: Descriptive Statistics

Let the number of test scores be:

$$n = 20.$$

The data in ascending order:

$$x_1 = 52, \quad x_2 = 62, \quad \ldots, \quad x_n = 100.$$

The range is:

$$r = x_n - x_1 = 100 - 52 = 48.$$

The mean is computed as:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = 84.5.$$

The standard deviation is given by:

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \bar{x}^2} = \frac{\sqrt{659}}{2} \approx 12.835.$$

The indices for the quartiles are:

$$\begin{aligned} i_{Q_1} &= \frac{n}{4} + \frac{1}{2} = 5.5, \\ i_{Q_2} &= \frac{n}{2} + \frac{1}{2} = 10.5, \\ i_{Q_3} &= \frac{3}{4}n + \frac{1}{2} = 15.5. \end{aligned}$$

Calculating the quartiles:

$$\begin{aligned} Q_1 &= \frac{x_{\lceil i_{Q_1} - \frac{1}{2} \rceil} + x_{\lfloor i_{Q_1} + \frac{1}{2} \rfloor}}{2} = \frac{x_5 + x_6}{2} = 77, \\ Q_2 &= \frac{x_{\lceil i_{Q_2} - \frac{1}{2} \rceil} + x_{\lfloor i_{Q_2} + \frac{1}{2} \rfloor}}{2} = \frac{x_{10} + x_{11}}{2} = 87, \\ Q_3 &= \frac{x_{\lceil i_{Q_3} - \frac{1}{2} \rceil} + x_{\lfloor i_{Q_3} + \frac{1}{2} \rfloor}}{2} = \frac{x_{15} + x_{16}}{2} = 94. \end{aligned}$$

The interquartile range (IQR) is:

$$\text{IQR} = Q_3 - Q_1 = 94 - 77 = 17.$$

Answer for Part 2:

$$Q_2 = 87, \quad \bar{x} = 84.5, \quad r = 48, \quad \sigma = \frac{\sqrt{659}}{2} \approx 12.835, \quad \text{IQR} = 17.$$

Part 3: Outlier Detection

Using the outlier rule:

$$Q_1 - 1.5 \times \text{IQR} = 77 - 1.5 \times 17 = 51.5,$$

$$Q_3 + 1.5 \times \text{IQR} = 94 + 1.5 \times 17 = 119.5.$$

Since all values in the dataset satisfy:

$$51.5 < x_1 < x_n < 119.5,$$

there are no outliers.

Answer: No outliers.