Question 4: Counting Five-Digit Numbers

1. How many five-digit numbers can be written?

Let the number of digits be:

$$K = 5.$$

The set of all possible numbers in one digit is:

$$D = \{0, 1, \ldots, 9\}.$$

The number of elements in D is:

$$d = \#D = 10.$$

Thus, the total number of five-digit numbers is:

$$n = d^K = 100\,000.$$

Other cases:

- Excluding numbers with the most significant digit 0:

$$n_{\setminus\{0\}} = (d-1)d^{K-1} = 90\,000.$$

- Including negative numbers:

$$n_{\mathbb{Z}} = 2d^K - 1 = 199\,999.$$

- Including negative numbers but excluding numbers with the most significant digit 0:

$$n_{\mathbb{Z}\setminus\{0\}} = 2(d-1)d^{K-1} = 180\,000.$$

Answer:

$$
\begin{aligned}
n &= 100\,000, \\
n_{\setminus\{0\}} &= 90\,000, \\
n_{\mathbb{Z}} &= 199\,999, \\
n_{\mathbb{Z}\setminus\{0\}} &= 180\,000.
\end{aligned}
$$

2. How many five-digit numbers contain at least one even digit?

Define the set of odd digits:

$$D_{\text{odd}} = \{1, 3, 5, 7, 9\}.$$

The number of elements in D_{odd} is:

$$d_{\text{odd}} = 5.$$

The number of five-digit numbers with only odd digits:

$$n_{\text{odd}} = d_{\text{odd}}^{K} = 3\,125.$$

Thus, the number of five-digit numbers containing at least one even digit:

$$n_{\#\{\text{even}\geq 1\}} = n - n_{\text{odd}} = 96\,875.$$

Other cases:

- Excluding numbers with the most significant digit 0:

$$n_{\setminus\{0\}_{\#\{\text{even}\geq 1\}}} = n_{\setminus\{0\}} - n_{\text{odd}} = 86\,875.$$

- Including negative numbers:

$$n_{\mathbb{Z}_{\#\{\text{even}\geq 1\}}} = n_{\mathbb{Z}} - n_{\text{odd}} = 196\,874.$$

- Including negative numbers but excluding numbers with the most significant digit 0:

$$n_{\mathbb{Z}\setminus\{0\}_{\#\{\text{even}\geq 1\}}} = n_{\mathbb{Z}\setminus\{0\}} - n_{\text{odd}} = 176\,875.$$

Answer:

$$n_{\#\{\text{even}\geq 1\}} = 96\,875,$$
$$n_{\setminus\{0\}_{\#\{\text{even}\geq 1\}}} = 86\,875,$$
$$n_{\mathbb{Z}_{\#\{\text{even}\geq 1\}}} = 196\,874,$$
$$n_{\mathbb{Z}\setminus\{0\}_{\#\{\text{even}\geq 1\}}} = 176\,875.$$