Question 2: European Roulette Probability Problems

Problem Statement

A *European roulette* consists of a wheel with 37 numbered pockets, labeled from 0 to 36. The numbers are distributed as described in the following parts.

Part 1: Probability of an Even Number Pocket

Let the probability space be defined by

$$\begin{aligned}
&\Omega = \{0, 1, 2, \ldots, 36\}, \\
&\mathcal{F} = \mathcal{P}(\Omega), \\
&\mathbb{P}: \quad \mathbb{P}(\{0\}) = \mathbb{P}(\{1\}) = \cdots = \mathbb{P}(\{36\}) = \frac{1}{37}.
\end{aligned}$$

Define the set of even-numbered pockets as

$$S_{\text{even}} = \{2, 4, 6, \ldots, 36\}.$$

Since there are 18 even numbers in Ω, we have

$$\mathbb{P}(S_{\text{even}}) = \frac{\#S_{\text{even}}}{\#\Omega} = \frac{18}{37}.$$

Answer for Part 1:

$$\boxed{\mathbb{P}(S_{\text{even}}) = \frac{18}{37}}$$

Part 2: Probability of a Red Pocket

Let the set of red pockets be

$$S_{\text{red}} = \{1, 3, 5, \ldots, 35\}.$$

Assuming there are 18 red pockets (as given), it follows that

$$\mathbb{P}(S_{\text{red}}) = \frac{\#S_{\text{red}}}{\#\Omega} = \frac{18}{37}.$$

Answer for Part 2:

$$\boxed{\mathbb{P}(S_{\text{red}}) = \frac{18}{37}}$$

Part 3: Conditional Probability of an Even Pocket Given a Red Outcome

We wish to compute the probability

$$\mathbb{P}_{S_{\text{red}}}(S_{\text{even}}) = \frac{\mathbb{P}(S_{\text{red}} \cap S_{\text{even}})}{\mathbb{P}(S_{\text{red}})}.$$

Let

$$S_{\text{red}} \cap S_{\text{even}} = \{12, 14, \ldots, 36\}.$$

Then, by counting the number of elements in this intersection relative to the total number of red pockets, we have

$$\mathbb{P}_{S_{\text{red}}}(S_{\text{even}}) = \frac{\#\{12, 14, \ldots, 36\}}{\#S_{\text{red}}} = \frac{4}{9}.$$

Answer for Part 3:

$$\boxed{\mathbb{P}_{S_{\text{red}}}(S_{\text{even}}) = \frac{4}{9}}$$