Question 2

You have three coins in your pocket, two fair ones but the third is biased with probability of heads p and tails $1 - p$. One coin selected at random drops on the floor, landing heads up. How likely is it that it is one of the fair coins?

Solution

Let *head* and *tail* denote the results of a drop. Define the probability space as

$$\begin{aligned} &\Omega = \{\text{head, tail}\} \\ &\mathcal{F} = \mathcal{P}(\Omega) \\ &\mathbb{P}: \ \text{the probability measure on } \mathcal{F} \end{aligned}$$

Let F and B denote the events of selecting a fair coin and a biased coin, respectively, then we have

$$\begin{aligned} \mathbb{P}_F(\{\text{head}\}) &= \frac{1}{2} \\ \mathbb{P}_B(\{\text{head}\}) &= p \end{aligned}$$

By applying the principle of symmetry, we have

$$\begin{aligned} \mathbb{P}(F) &= \frac{2}{3} \\ \mathbb{P}(B) &= \frac{1}{3} \end{aligned}$$

We know that $\{F, B\}$ is a partition of Ω, then by applying the law of total probability, we have

$$\mathbb{P}(\{\text{head}\}) = \mathbb{P}_F(\{\text{head}\})\mathbb{P}(F) + \mathbb{P}_B(\{\text{head}\})\mathbb{P}(B) = \frac{p+1}{3}$$

Then by applying the Bayes' Theorem, we have

$$\mathbb{P}_{\{\text{head}\}}(F) = \frac{\mathbb{P}_F(\{\text{head}\})\mathbb{P}(F)}{\mathbb{P}(\{\text{head}\})} = \frac{1}{p+1}$$

Answer

$$\boxed{\mathbb{P}_{\{\text{head}\}}(F) = \frac{1}{p+1}}$$