Question 3

Three fair dice are thrown. Let X denote the number of dice that land with the same number of dots. Describe the probability distribution function of X.

Solution

By applying the principle of symmetry, easy to define the probability space as

$$\begin{aligned}
&\Omega = \{1, 2, \ldots, 6\} \\
&\mathcal{F} = \mathcal{P}(\Omega) \\
&\mathbb{P}: \ \mathbb{P}(\{1\}) = \mathbb{P}(\{2\}) = \cdots = \mathbb{P}(\{6\}) = \frac{1}{6}
\end{aligned}$$

Then as given, we have

$$X: \ \Omega \to R, \quad R = \{1, 2, 3\}$$

Then we have the *pdf* of X

$$p(x) = \begin{cases} \dfrac{6 \times 5 \times 4}{6^3} = \dfrac{5}{9}, & x = 1 \\ \dfrac{6 \times 3 \times 5}{6^3} = \dfrac{5}{12}, & x = 2 \\ \dfrac{6}{6^3} = \dfrac{1}{36}, & x = 3 \end{cases}$$

Or a general formula, with $N_i \in \{1, 2, 3\}$, $i \in \{1, 2, \ldots, 6\}$ denotes the absolute frequency of every face (*i.e.* $N_1 = 2$ means that face 1 shows 2 times)

$$p(x) = \frac{1}{6^3} \sum_{\sum_{i=1}^{6} N_i = 3} \frac{3!}{N_1! N_2! \ldots N_6!}, \quad \max(\{N_1, N_2, \ldots, N_6\}) = x, \quad x \in R$$

Answer

$$\boxed{p(x) = \begin{cases} \dfrac{5}{9}, & x = 1 \\ \dfrac{5}{12}, & x = 2 \\ \dfrac{1}{36}, & x = 3 \end{cases}}$$