Question 2

An urn contains six balls, numbered 1 to 6. Two balls are drawn at random, without replacement. Let X be the difference between the larger and smaller of the two numbers drawn. What is the probability distribution of X?

Solution

By applying the principle of symmetry, easy to define the probability space as

$$\begin{aligned}
&\Omega = \{1, 2, \ldots, 6\} \\
&\mathcal{F} = \mathcal{P}(\Omega) \\
&\mathbb{P}: \ \mathbb{P}(\{1,2\}) = \mathbb{P}(\{1,3\}) = \cdots = \mathbb{P}(\{6,5\}) = \frac{1}{6} \times \frac{1}{5} = \frac{1}{30}
\end{aligned}$$

Then we have the *pdf* of X

$$p(x) = \begin{cases}
\mathbb{P}(X=1) = \mathbb{P}(\{1,2\},\{2,1\},\ldots,\{6,5\}) & = \dfrac{5}{15} \approx 0.333, \qquad x = 1 \\
\mathbb{P}(X=2) = \mathbb{P}(\{1,3\},\{2,4\},\ldots,\{6,4\}) & = \dfrac{4}{15} \approx 0.267, \qquad x = 2 \\
\mathbb{P}(X=3) = \mathbb{P}(\{1,4\},\{2,5\},\ldots,\{6,3\}) & = \dfrac{3}{15} = 0.2, \qquad\quad x = 3 \\
\mathbb{P}(X=4) = \mathbb{P}(\{1,5\},\{2,6\},\ldots,\{6,2\}) & = \dfrac{2}{15} \approx 0.133, \qquad x = 4 \\
\mathbb{P}(X=5) = \mathbb{P}(\{1,6\},\{6,1\}) & = \dfrac{1}{15} \approx 0.067, \qquad x = 5
\end{cases}$$

Or a general formula without cases

$$p(x) = \frac{6-x}{\binom{6}{2}} = \frac{6-x}{15}, \quad x \in \{1, 2, \ldots, 5\}$$

Answer

$$\boxed{p(x) = \begin{cases}
\dfrac{5}{15} \approx 0.333, & x = 1 \\
\dfrac{4}{15} \approx 0.267, & x = 2 \\
\dfrac{3}{15} = 0.2, & x = 3 \\
\dfrac{2}{15} \approx 0.133, & x = 4 \\
\dfrac{1}{15} \approx 0.067, & x = 5
\end{cases}}$$