

Question 1

Eight thousand lottery tickets are sold for \$5 each.

One ticket will win \$2,000, two tickets will win \$750 each, and five tickets will win \$100 each.

Part 1: Let X denote the net gain from the purchase of a randomly selected ticket. Construct the probability distribution of X .

Solution

Easy to get the *pdf* of X

$$p(x) = \begin{cases} \frac{1}{8000}, & x = 2000 - 5 = 1995 \\ \frac{2}{8000}, & x = 750 - 5 = 745 \\ \frac{5}{8000}, & x = 100 - 5 = 95 \\ \frac{8000 - 1 - 2 - 5}{8000} = \frac{7992}{8000}, & x = 0 - 5 = -5 \end{cases}$$

Answer

$$p(x) = \begin{cases} \frac{1}{8000}, & x = 1995 \\ \frac{2}{8000}, & x = 745 \\ \frac{5}{8000}, & x = 95 \\ \frac{7992}{8000}, & x = -5 \end{cases}$$

Part 2: Compute the expected value μ of X and interpret its meaning.

Solution

As given, easy to get

$$\begin{aligned} \mu &= 1995p(1995) + 745p(745) + 95p(95) + (-5)p(-5) \\ &= \frac{399}{1600} + \frac{149}{800} + \frac{19}{320} - \frac{999}{200} \\ &= -\frac{9}{2} \end{aligned}$$

Answer

$$\mu = -\frac{9}{2}$$

Part 3: Compute the standard deviation σ of X .

Solution

Easy to get

$$\begin{aligned}\sigma &= \sqrt{-\mu^2 + \mathbb{E}[X^2]} \\&= \sqrt{-\mu^2 + (1\,995^2 p(1\,995) + 745^2 p(745) + 95^2 p(95) + (-5)^2 p(-5))} \\&= \sqrt{-\frac{81}{4} + \left(\frac{398\,0025 + 111\,0050 + 45\,125 + 199\,800}{8\,000} \right)} \\&= \sqrt{\frac{5\,173}{8}} \\&\approx 25.429\end{aligned}$$

Answer

$$\sigma = \sqrt{\frac{5\,173}{8}} \approx 25.429$$