Question 2

Assume that the random variable X has the following probability mass function:

$$p(x) = \begin{cases} \frac{1}{8}, & x = -1 \\ \frac{1}{4}, & x = 0 \\ \frac{1}{2}, & x = 1 \\ \frac{1}{8}, & x = 2 \end{cases}$$

Part 1: Compute the expectation of X and of X^2.

Solution

Easy to get

$$\begin{aligned} \mathbb{E}[X] &= -1p(-1) + 0p(0) + 1p(1) + 2p(2) \\ &= -\frac{1}{8} + 0 + \frac{1}{2} + \frac{1}{4} \\ &= \frac{5}{8} \end{aligned}$$

As well as

$$\begin{aligned} \mathbb{E}\left[X^2\right] &= (-1)^2p(-1) + 0^2p(0) + 1^2p(1) + 2^2p(2) \\ &= \frac{1}{8} + 0 + \frac{1}{2} + \frac{1}{2} \\ &= \frac{9}{8} \end{aligned}$$

Answer

$$\boxed{\begin{aligned} \mathbb{E}[X] &= \frac{5}{8} \\ \mathbb{E}\left[X^2\right] &= \frac{9}{8} \end{aligned}}$$

Part 2: Compute the variance of X and of X^2.

Solution

Easy to get the variance of X

$$\begin{aligned} V(X) &= \mathbb{E}\left[X^2\right] - (\mathbb{E}[X])^2 \\ &= \frac{9}{8} - \left(\frac{5}{8}\right)^2 \\ &= \frac{47}{64} \end{aligned}$$

As well as the variance of X^2

$$\begin{aligned} V(X^2) &= -\left(\mathbb{E}\left[X^2\right]\right)^2 + \mathbb{E}\left[\left(X^2\right)^2\right] \\ &= -\frac{81}{64} + \left((-1)^4 p(-1) + 0^4 p(0) + 1^4 p(1) + 2^4 p(2)\right) \\ &= -\frac{81}{64} + \left(\frac{1}{8} + 0 + \frac{1}{2} + 2\right) \\ &= \frac{87}{64} \end{aligned}$$

Answer

$$\boxed{\begin{aligned} V(X) &= \frac{47}{64} \\ V(X^2) &= \frac{87}{64} \end{aligned}}$$